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THEORY OF LIE-DERIVATIVES AND MOTIONS IN TACHIBANA SPACES

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Abstract. In this paper, we have determined the expression of the Lie-derivatives of the metric tensor and connection parameters of the Tachibana spaces and its Subspace in the complex co-ordinate system. Further, we have studied the properties of motion and Affine motion in Tachibana space.

1. Introduction. Suppose C_n is an n-dimensional complex space referred to an allowable co-ordinate system

$$(Z^i, Z^{\overline{i}}) = (Z^1, Z^2, Z^3, \dots, Z^n; \quad Z^{\overline{1}}, Z^{\overline{2}}, Z^{\overline{3}}, \dots, Z^{\overline{n}}) \quad (i = 1,2,3,4\dots,n; \quad \overline{i} = \overline{1},\overline{2},\overline{3},\overline{4},\dots,\overline{n})$$

We introduce in C_n, a metric defined by the positive definite Hermitian form [Yano & Bochner 1953]

$$ds^2 = 2gi\overline{j}(Z^i, Z^{\overline{i}})dZ^i dZ^{\overline{j}} \tag{1.1}$$

If the tensor $g_{i\overline{j}}$ also satisfies the Kaehler's condition [Yano & Bochner 1953].

$$\frac{\partial g_{i\overline{j}}}{\partial Z^{\overline{k}}} = \frac{\partial g_{i\overline{k}}}{\partial Z^{\overline{j}}}, \tag{1.2}$$

then the complex space C_n is called a Kaehlerian space. If this space also satisfies the condition

$$F^h_{j,i} = 0, \tag{1.3}$$

then the space is called a Tachibana space and is denoted by T^C_n. Consider a subspace T^C_m of T^C_n given by equations

$$\left.\begin{aligned} Z^i &= Z^i(u^\alpha); \quad \alpha = 1,2,3,\dots,m; \\ Z^{\overline{i}} &= Z^{\overline{i}}(u^{\overline{\alpha}}); \quad \overline{\alpha} = \overline{1},\overline{2},\overline{3},\dots,\overline{m} \end{aligned}\right\} \tag{1.4}$$

The components $g_{\alpha\overline{\beta}}$ of the fundamental metric tensor of T^C_m are given by

$$g_{\alpha\overline{\beta}} = g_{i\overline{j}} p^i_\alpha p^{\overline{j}}_{\overline{\beta}},$$

where $p^i_\alpha = \partial Z^i / \partial u^\alpha$ and $p^{\overline{i}}_{\overline{\alpha}} = \partial Z^{\overline{i}} / \partial u^{\overline{\alpha}}$.

2. Lie-Derivatives in Tachibana Space T^C_n. Consider an infinitesimal transformation of the type

$$^1Z^h = Z^h + v^h(Z, \overline{Z})dT_{\text{caps}}, \tag{2.1}$$

where Z^h and $^1Z^h$ are the complex co-ordinates of Tachibana manifolds T^C_n and $^1T^C_n$ respectively, $v^h(Z, \overline{Z})$ are the contravariant components of a vector field along which the deformation is considered and dT_{caps} is an infinitesimal parameter. The corresponding variation in $Z^{\overline{h}}$ is given by

$$^1Z^{\overline{h}} = Z^{\overline{h}} + v^{\overline{h}}(Z, \overline{Z})dT_{\text{caps}}. \tag{2.2}$$

Differentiating equation (2.1) w.r. to Z^j and $Z^{\overline{j}}$ respectively, we get

$$\partial_j {}^1Z^h = \delta^h_j + \partial_j v^h dT_{\text{caps}} \tag{2.3}$$

and $\partial_{\overline{j}}{}^1Z^h = \delta^h_{\overline{j}} v^h dT\text{caps},$ (2.4)

where $\partial_j = \dfrac{\partial}{\partial Z^j}$ and $\partial_{\overline{j}} = \dfrac{\partial}{\partial Z^{\overline{j}}}$.

Rewriting (2.1) as $Z^h = {}^1Z^h - v^h(Z,\overline{Z})dT\text{caps}$ and differentiating with respect to ${}^1Z^h$ and using $\partial_j v^h dT\text{caps} = {}^1\partial_j v^h dT\text{caps}$, $\left({}^1\partial_j = \frac{\partial}{\partial {}^1Z^j}\right)$, we have

$${}^1\partial_j Z^h = \delta^h_j - \delta_j v^h dT_{\text{caps}} \tag{2.5}$$

Let $Z^{k\overline{l}}(Z,\overline{Z})$ be a tensor field defined over T^C_n, the value of the function $Z^{k\overline{l}}(Z,\overline{Z})$ at ${}^1Z$ (using point transformation) can be written (by Taylor's expansion theorem)

$$Z^{k\overline{l}}({}^1Z,{}^1\overline{Z}) = Z^{k\overline{l}}(Z,\overline{Z}) + [v^j\partial_j Z^{k\overline{l}} + v^{\overline{j}}\partial_{\overline{j}} Z^{k\overline{l}}]dT_{\text{caps}}, \tag{2.6}$$

in which only first order of dT_{caps} has been considered. Using

$$d^U Z^{k\overline{l}} \text{ (by def.)} = Z^{k\overline{l}}({}^1Z,{}^1\overline{Z}) - Z^{k\overline{l}}(Z,\overline{Z}), \tag{2.7}$$

we have

$$d^U Z^{k\overline{l}} = [v^j\partial_j Z^{k\overline{l}} + v^{\overline{j}}\partial_{\overline{j}} Z^{k\overline{l}}]dT_{\text{caps}}. \tag{2.8}$$

Let us consider the equation (2.1) as the co-ordinate transformation and ${}^1Z^{k\overline{l}}({}^1Z,{}^1\overline{Z})$ as the components of the tensor field $Z^{k\overline{l}}(Z,\overline{Z})$ in the ${}^1Z$ co-ordinate system. This will give

$${}^1Z^{k\overline{l}}({}^1Z,{}^1\overline{Z}) = Z^{mp}\partial_m {}^1Z^k \partial_p {}^1Z^{\overline{l}} + Z^{m\overline{p}}\partial_m {}^1Z^k \partial_{\overline{p}} {}^1Z^{\overline{l}} + Z^{\overline{m}p}\partial_{\overline{m}} {}^1Z^k \partial_p {}^1Z^{\overline{l}} + Z^{\overline{m}\overline{p}}\partial_{\overline{m}} {}^1Z^k \partial_{\overline{p}} {}^1Z^{\overline{l}}. \tag{2.9}$$

On further calculations with the help of equation (2.3) and (2.4), we get

$${}^1Z^{k\overline{l}}({}^1Z,{}^1\overline{Z}) = Z^{k\overline{l}} + [Z^{kp}\partial_p v^{\overline{l}} + Z^{k\overline{p}}\partial_{\overline{p}} v^{\overline{l}} + Z^{m\overline{l}}\partial_m v^k + Z^{\overline{m}\overline{l}}\partial_{\overline{m}} v^k]dT_{\text{caps}}. \tag{2.10}$$

We define

$$d^m Z^{k\overline{l}} \text{ (by def.)} = {}^1Z^{k\overline{l}} - Z^{k\overline{l}} = [Z^{kp}\partial_p v^{\overline{l}} + Z^{k\overline{p}}\partial_{\overline{p}} v^{\overline{l}} + Z^{m\overline{l}}\partial_m v^k + Z^{\overline{m}\overline{l}}\partial_{\overline{m}} v^k]dT_{\text{caps}} \tag{2.11}$$

Now, the Lie-derivative of the tensor field $Z^{k\overline{l}}(Z,\overline{Z})$ in T^C_n is defined (Rund [1959]), Yano [1957]) as

$$£_v Z^{k\overline{l}}(Z,\overline{Z}) \text{ (by def.)} = \frac{d^u Z^{k\overline{l}} - d^m Z^{k\overline{l}}}{dT_{\text{caps}}}, \tag{2.12}$$

which after the application of equations (2.8), (2.11) and interchange of some indices gives

$$£_v Z^{k\overline{l}}(Z,\overline{Z}) = v^j\partial_j Z^{k\overline{l}} + v^{\overline{j}}\partial_{\overline{j}} Z^{k\overline{l}} - Z^{k\overline{j}}\partial_{\overline{j}} v^{\overline{l}} - Z^{k\overline{j}}\partial_{\overline{j}} v^{\overline{l}} - Z^{j\overline{l}}\partial_{\overline{j}} v^k - Z^{\overline{j}\overline{l}}\partial_{\overline{j}} v^k. \tag{2.13}$$

The Lie-derivatives of covariant tensor $Z_{k\overline{l}}(Z,\overline{Z})$ and a scalar $N(Z,\overline{Z})$ can be obtained similarly

$$£_v Z_{k\overline{l}}(Z,\overline{Z}) = v^j\partial_j Z_{k\overline{l}} + v^{\overline{j}}\partial_{\overline{j}} Z_{k\overline{l}} + Z_{k\overline{j}}\partial_{\overline{l}} v^j + Z_{k\overline{j}}\partial_{\overline{l}} v^{\overline{j}} + Z_{j\overline{l}}\partial_k v^j + Z_{\overline{j}\overline{l}}\partial_k v^{\overline{j}} \tag{2.14}$$

and $£_v N(Z,\overline{Z}) = v^j\partial_j N + \partial_{\overline{j}} v^{\overline{j}} N.$ (2.15)

As a consequence of equations (2.13) and (2.14), the Lie-derivatives of the components of the fundamental tensors $g^{i\overline{j}}(Z,\overline{Z})$ and $g_{i\overline{j}}(Z,\overline{Z})$ are given by

$$£_v g^{i\overline{j}}(Z,\overline{Z}) = v^k\partial_k g^{i\overline{j}} + v^{\overline{k}}\partial_{\overline{k}} g^{i\overline{j}} - g^{ik}\partial_k v^{\overline{j}} - g^{i\overline{k}}\partial_{\overline{k}} v^{\overline{j}} - g^{k\overline{j}}\partial_k v^i - g^{\overline{k}\overline{j}}\partial_{\overline{k}} v^i \tag{2.16}$$

and $£_v g_{i\overline{j}}(Z,\overline{Z}) = v^k\partial_k g_{i\overline{j}} + v^{\overline{k}}\partial_{\overline{k}} g_{i\overline{j}} - g_{ik}\partial_{\overline{j}} v^{\overline{k}} + g_{i\overline{k}}\partial_{\overline{j}} v^{\overline{k}} + g_{k\overline{j}}\partial_i v^k + g_{\overline{k}\overline{j}}\partial_{\overline{i}} v^{\overline{k}}$ (2.17)

respectively.

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THE SIR MODEL OF CONTAGIOUS ILLNESS HAS NOT PERIODIC ORBIT

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Abstract. In this paper we prove that there is not any period T for the contagious illnesses, which follows from susceptibles, infectives and removed (SIR) model.

Introduction. There are some contagious illnesses that are very dangerous for human beings, The knowledge is developing. At the same time we hope to find sorpe solutions for preventing from them or we wish to get some information about them that can help us to protect society. Let us have an overview on SIR model [1,2,3]. This model can give us many suitable information about the contagious illness in a densely society like a city or a school.

The assumptions for this model are:

(i) The total population is constant The epidemic does not affect on the number of population;

(ii) The population is well stirred. Each member has an equal chance to meet the others;

(iii) Any individual in the population who got the illness either obtained immunity or died.

We denote the number of susceptible people and infective people by $S(t)$ and $I(t)$ respectively. Also the number of removed people will be shown with $R(t)$ in time t. Removed people are the individuals who died or obtained immunity after getting the disease. We assume that susceptible people can inter into the infections and the infected people move into the removed people. We must pay attention that if the population of susceptible is big, the number of people who inter into the infection will increase in unit time. Denoting the infection rate by $r > 0$. In the same way the number of infected people can affect the number of removed people. Denoting the recovery rate by $a > 0$. By using the suitable proportions the mathematical model of SIR is

$$\begin{cases} \dot{S} = -rSI \\ \dot{I} = rSI - aI \\ \dot{R} = aI \end{cases}$$

We assume $\dot{S} + \dot{I} + \dot{R} = 0$. It is the same as the first condition and means the total population is constant. Also the following initial conditions are considerable $S(0) > 0$, $I(0) > 0$, $R(0) = 0$ and $I(0) + S(0) = N$ where N is the total population.

In the next section we show that there is no any period T for the illness i.e. we cannot predict when the illness comes back with a regular period.

The Contagious Illness is not Periodic. To prove the SIR model has not periodic orbit we assume that $S(t)$, $I(t)$ and $R(t)$ are periodic solutions for it with the period T and then by the Fourier form [4] of periodic functions we will find a contradiction.

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Since $S(t)$, $I(t)$ and $R(t)$ are assumed to be periodic with the period T then we have

$$\begin{cases} S(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t) & \text{(I)} \\ I(t) = \frac{a'_0}{2} + \sum_{n=1}^{\infty} a'_n \cos(n\omega t) + \sum_{n=1}^{\infty} b'_n \sin(n\omega t) & \text{(II)} \\ I(t) = \frac{a''_0}{2} + \sum_{n=1}^{\infty} a''_n \cos(n\omega t) + \sum_{n=1}^{\infty} b''_n \sin(n\omega t) & \text{(III)} \end{cases}$$

where $\omega = \frac{2\pi}{T}$ and $a'_n, b'_n, a''_n, b''_n, a_n, b_n \in \mathbb{R}$.

With respect to $\dot{S} = -\dot{I} - \dot{R}$ we have

$$\sum_{n=1}^{\infty} nb'_n \omega \cos(n\omega t) - \sum_{n=1}^{\infty} na'_n \omega \sin(n\omega t) = -\sum_{n=1}^{\infty} nb_n \omega \cos(n\omega t) + \sum_{n=1}^{\infty} na_n \omega \sin(n\omega t) - \sum_{n=1}^{\infty} nb''_n \omega \cos(n\omega t) + \sum_{n=1}^{\infty} na''_n \omega \sin(n\omega t)$$

By putting $t = 0$ in above we get

$$\sum_{n=1}^{\infty} nb'_n \omega = -\sum_{n=1}^{\infty} nb_n \omega - \sum_{n=1}^{\infty} nb''_n \omega \quad \text{or} \quad \sum_{n=1}^{\infty} nb''_n + \sum_{n=1}^{\infty} nb'_n = -\sum_{n=1}^{\infty} nb_n \qquad (*)$$

Now we want to find the period T, which reveals us how long the time takes for coming back the illness. Equations (1), (I) and (II) imply

$$\begin{aligned} &\sum_{n=1}^{\infty} nb_n \omega \cos(n\omega t) - \sum_{n=1}^{\infty} na_n \omega \sin(n\omega t) \\ &= -r\Bigg[\frac{a_0}{2}\frac{a'_0}{2} + \sum_{n=1}^{\infty} \frac{a_0}{2} a'_n \cos(n\omega t) + \sum_{n=1}^{\infty} \frac{a_0}{2} b'_n \sin(n\omega t) + \sum_{n=1}^{\infty} \frac{a'_0}{2} a_n \cos(n\omega t) \\ &\quad + \sum_{n=1}^{\infty} a'_n \cos(n\omega t) \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b'_n \sin(n\omega t) \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} \frac{a'_0}{2} b_n \sin(n\omega t) \\ &\quad + \sum_{n=1}^{\infty} a'_n \cos(n\omega t) \sum_{n=1}^{\infty} b_n \sin(n\omega t) + \sum_{n=1}^{\infty} b'_n \sin(n\omega t) \sum_{n=1}^{\infty} b_n \cos(n\omega t)\Bigg] \end{aligned}$$

By putting $t = 0$ in above we get

$$\sum_{n=1}^{\infty} nb_n \omega = -r\left[\frac{a_0}{2}\frac{a'_0}{2} + \sum_{n=1}^{\infty} \frac{a_0}{2} a'_n + \sum_{n=1}^{\infty} \frac{a'_0}{2} a_n + \sum_{n=1}^{\infty} a'_n + \sum_{n=1}^{\infty} a_n\right]$$

Thus
$$T = \frac{2\pi \sum_{n=1}^{\infty} nb_n}{-r\left[\frac{a_0 a'_0}{4} + \sum_{n=1}^{\infty} \frac{a_0}{2} a'_n + \sum_{n=1}^{\infty} \frac{a'_0}{2} a_n + \sum_{n=1}^{\infty} a'_n \sum_{n=1}^{\infty} a_n\right]} \qquad (**)$$